Optimal control with weighted average costs and temporal logic specifications

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Motivation

Goal:
- Optimal control for an autonomous system doing a complex task

Challenges:
- Reasoning about how system properties change over time
- Specifying properties like safety, response, priority, liveness, and persistence
- Optimizing the system trajectory to conserve fuel or minimize time
Problem Description

Simple example:

- **Task**: repeatedly visit PICKUP

**Given:**
- **System model**: transition system $\mathcal{T}$ with costs and weights
- **Task specification**: linear temporal logic (LTL) formula $\varphi$

**Problem:** Minimize

$$J(\sigma) := \lim_{n \to \infty} \sup \frac{\sum_{i=0}^{n} c(\sigma_i, \sigma_{i+1})}{\sum_{i=0}^{n} w(\sigma_i, \sigma_{i+1})}$$

over all system trajectories $\sigma$ that satisfy the LTL specification $\varphi$. 

![Diagram showing a transition system with states START, PICKUP, 1, 2, 3, 4, and transitions with labels (0,1), (10,1), (fuel,time), and (1,1)]
Solution overview

1. Construct the product automaton $\mathcal{P} = \mathcal{T} \times \mathcal{A}_\varphi$

\[
P = \begin{array}{c}
\begin{array}{c}
\text{1} \\
\text{2} \\
\text{3} \\
\text{4}
\end{array} \\
\begin{array}{c}
(0,1) \\
(1,1) \\
(10,1)
\end{array}
\end{array} \quad \quad || \quad \begin{array}{c}
\text{q0} \\
\text{q1}
\end{array}
\]

\[
\varphi = \text{repeatedly visit PICKUP}
\]
Solution overview

1. Construct the product automaton $\mathcal{P} = \mathcal{T} \times \mathcal{A}_\varphi$

\[
\mathcal{P} = \begin{array}{c}
\{\text{START}\} \\
1 \\
\downarrow (0,1) \\
4 \\
\downarrow (1,1) \\
3 \\
\uparrow (10,1) \\
2 \\
\end{array} \\
\begin{array}{c}
\{\text{PICKUP}\} \\
\end{array}
\]

2. Show that problem is equivalent to finding an optimal cycle in $\mathcal{P}$

- $\sigma = \sigma_{\text{pre}}\sigma_{\text{suf}}$
- $J(\sigma_{\text{pre}}\sigma_{\text{suf}}^\omega) = J(\sigma_{\text{suf}}^\omega)$
- $\sigma_{\text{suf}}$ is a cycle including an accepting state in $\mathcal{P}$
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1. Construct the product automaton $P = \mathcal{T} \times A_\varphi$

$$P = \begin{array}{c}
\{\text{START}\} \\
1 \\
(0,1) \\
4 \\
(1,1) \\
3 \\
(10,1) \\
2 \\
(10,1) \\
\{\text{PICKUP}\}
\end{array} \ || \ \begin{array}{c}
\phi = \text{repeatedly visit PICKUP}
\end{array}$$

2. Show that problem is equivalent to finding an optimal cycle in $P$

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3. Compute optimal cycle using dynamic programming
Example—autonomous driving

Task: repeatedly visit $a$, $b$, and $c$ and avoid obstacles $x$

LTL spec: $\varphi = \Box \Diamond a \land \Box \Diamond b \land \Box \Diamond c \land \Box \neg x$

Figure: Driving task, with optimal run (blue) and feasible run (red).
Example—autonomous driving

**Task:** repeatedly visit $a$, $b$, and $c$ and avoid obstacles $x$

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**Figure:** Driving task, with optimal run (blue) and feasible run (red).

**Cost:** \( J_{\text{opt}} = 49 \) and \( J_{\text{feas}} = 71 \) (units)

**CPU time:** \( t_{\text{opt}} = 2.5 \) and \( t_{\text{feas}} = 0.68 \) (sec)